On model based seasonal adjustment procedures

Jörg Breitung[1]

Humboldt-Universität zu Berlin

Institut für Statistik und Ökonometrie

Spandauer Strasse 1 D-10178 Berlin

ABSTRACT: In this note the unobserved component approach underlying the software package SEATS is compared with the Beveridge-Nelson type of decomposition for seasonal time series. The main strength of the SEATS approach lies in the appealing model formulation and the careful specification and adjustment of the input series. However, there are some theoretical problems with orthogonal decompositions which may be avoided by using the Beveridge-Nelson approach. The German unemployment series is studied to illustrate the properties of the alternative methods practically.

1 Introduction

In recent years, model based seasonal adjustment procedures were suggested to overcome the ad-hoc character of widely used filter based procedures like CENSUS X-11. Most approaches rely on stochastic components using an ARIMA framework due to Box and Jenkins (1970). For example, Box et al. (1978), Nerlove et al. (1979), Harvey & Todd (1983) and Maravall & Pierce (1987), adopt an unobserved components framework with orthogonal components. Since the development of the software package SEATS (Gomez & Maravall, 1996), this approach becomes increasingly popular in practice.[2] Another approach based on

[1]The author likes to thank Victor Gomez for a helpful discussion of various problems related to the BN-decomposition. However, all remaining errors and misunderstandings are my own.

[2]In fact the TRAMO–SEATS software has the potential to become an international standard for seasonal adjustment like the Census X-11 procedure in former years. It can efficiently be used for routine application to a large number of series, as is done, for example at EUROSTAT.

an ARIMA framework makes use of a Beveridge & Nelson (1981) type of decomposition for nonstationary time series (Breitung, 1994; Newbold & Vougas, 1995; Hylleberg & Pagan, 1997).

In a recent paper Stier (1996) discusses the merits and pitfalls of the unobserved components approach underlying the software SEATS. As pointed out by Maravall & Feldmann (1997) some of the problems are resolved in more recent versions of the TRAMO–SEATS software. Other conceptual problems have a more fundamental origin and are involved by other seasonal adjustment procedures as well (e.g. Maravall & Feldmann 1997). This note addresses some further issues by comparing the unobserved component approach of SEATS with the Beveridge-Nelson (BN) type of decomposition.

It is fair to say that the recent work on the BN-decomposition does not directly aim at developing a new seasonal adjustment procedure with a broad practical scope. The extraction of the seasonal component from the series is only one possible application of the BN-decomposition. For example, Pagan & Hylleberg (1997) and Breitung & Franses (1998) employ the BN-decomposition for testing against seasonal unit roots. Whether the BN-decomposition has the potential to be a serious competitor for existing seasonal adjustment procedures remains to be seen. At a minimum this would require a thorough consideration of practical problems like the adjustment of outliers and working days as well as a powerful model selection procedure. This is, however, beyond the scope of the present work.

2 The BN-decomposition for seasonal time series

As in Maravall & Pierce (1987) and Stier (1996) consider a simple seasonal model given by

$$(1 - B^2)X_t = a_t \ , \tag{1}$$

where X_t ($t = 1, 2, \ldots, T$) is the observed time series, B denotes the backshift operator defined as $B^k X_t = X_{t-k}$, and a_t is assumed to be white noise with $E(a_t) = 0$ and $E(a_t^2) = \sigma_a^2$. The lag polynomial $(1 - B^2) = (1 - B)(1 + B)$ has two unit roots implying poles in the power spectrum at frequencies 0 and π. The pole at frequency zero reflects the trend behaviour of the time series, whereas the pole at frequency π represents the seasonal pattern. The idea of the BN-decomposition is to define two different components with a spectral peak at frequency 0 and π, respectively. This formal decomposition gives in turn rise to a trend and seasonal component.

For the simple model (1) the decomposition may be derived as follows[3]:

$$\begin{aligned} X_t &= (1-B^2)^{-1}a_t \\ &= \phi_1(1-B)^{-1}a_t + \phi_2(1+B)^{-1}a_t \\ &\equiv p_t + s_t \ , \end{aligned}$$

where $p_t = \phi_1(1-B)^{-1}a_t = \phi_1 \sum_{i=0}^{t-1} a_i$ is the *trend component*, $s_t = \phi_2(1+B)^{-1}a_t = \phi_2 \sum_{i=0}^{t-1}(-1)^i a_{t-i}$ is the seasonal component. Obviously, the trend component, p_t, is a random walk and the spectrum of s_t has a single pole at frequency π.[4]

The values of ϕ_1 and ϕ_2 are obtained from $(1-B^2)X_t = \phi_1(1+B) + \phi_2(1-B)a_t = a_t$ as $\phi_1 = \phi_2 = 1/2$. With this results the trend and seasonal components are given by

$$p_t = (1/2)(1+B)X_t \ , \tag{2}$$
$$s_t = (1/2)(1-B)X_t \ , \tag{3}$$

and, thus, the components result from applying simple one-sided filters to X_t.

As noted by Hylleberg & Pagan (1997) this decomposition admits an interpretation of evolving seasonals given by

$$X_t = \gamma_{0t} + \gamma_{1t}\cos(\pi t),$$

where the coefficients γ_{0t} and γ_{1t} follow random walk sequences with

$$\begin{aligned} \gamma_{0t} &= \gamma_{0,t-1} + v_{0t} \ , \\ \gamma_{1t} &= \gamma_{1,t-1} + v_{1t} \ , \end{aligned}$$

where $v_{0t} = a_t/2$ and $v_{1t} = (-1)^t a_t/2$.

The results can be generalized to models of the form

$$(1-B^s)X_t = \mu + \psi(B)a_t \tag{4}$$

[3]Note that $(1-B)$ and $(1+B)$ have roots on the unit circle of the complex plane. As a consequence, the inverse of such a polynomial does not exist. However, it is possible to adopt a different definition of the inverse of the polynomial, which can be applied in our case (cf. Gregoir & Laroque, 1993).

[4]For more general models as (1), the decomposition involves an additional transitory component (cf Breitung, 1994).

or

$$(1-B)(1-B^s)X_t = \mu + \psi(B)a_t \ , \tag{5}$$

where s is the seasonal frequency (e.g. $s = 12$ for monthly series) and $\psi(B)$ is a (possibly infinite) lag polynomial, which may result from a ARMA$(p,q)(P,Q)_s$ model in the terminology of Box and Jenkins (1970). The corresponding BN-decompositions for such models are given in Breitung (1994) and Newbold & Vougas (1995).

3 The unobserved component approach

Time series models based on unobserved components have a long tradition and were considered by Nerlove et al. (1979), Harvey & Todd (1983), Maravall & Pierce (1987) among others. The appealing feature of this kind of models is that the components are assumed to be mutually uncorrelated. It is often argued that trend and seasonal behaviour have a quite different origin (e.g. Maravall & Feldmann, 1997, p. 205) so that it is implausible to specify the components as in the BN-decomposition, where all components are driven by the same innovations.

With respect to the simple model (1) it may therefore be preferable to construct an orthogonal decomposition as

$$\begin{aligned} X_t &= (1-B)^{-1}u_t + (1+B)^{-1}v_t \\ &\equiv p_t^* + s_t^*, \end{aligned} \tag{6}$$

where u_t and v_t are uncorrelated white noise sequences with $E(u_t^2) = \sigma_u^2$ and $E(v_t^2) = \sigma_v^2$. We have

$$E(a_t^2) = 2\sigma_u^2 + 2\sigma_v^2$$
$$E(a_t a_{t-1}) = \sigma_u^2 - \sigma_v^2 = 0$$

and, therefore,

$$\sigma_u^2 = \sigma_v^2 = \sigma_a^2/4.$$

The minimum mean squared error estimators for p_t^* and s_t^* are obtained as (e.g. Stier, 1996)

$$\hat{p}_t^* = (1/4)(B^{-1} + 2 + B)X_t \tag{7}$$
$$\hat{s}_t^* = (1/4)(-B^{-1} + 2 - B)X_t \ , \tag{8}$$

where B^{-1} is the forward operator. Note that the resulting filters are "squared versions" of the filters $\Psi_0(B) = (1/2)(1+B)$ and $\Psi_\pi(B) = (1/2)(1-B)$ for the BN-decomposition, in the sense that they are obtained as $\Psi_0(B^{-1})\Psi_0(B)$ and $\Psi_\pi(B^{-1})\Psi_\pi(B)$, respectively.

The resulting filters have the advantage that they are symmetric and, thus, do not introduce a phase shift. However, an important problem with such filters is that the corresponding characteristic equations have multiple unit roots. As a consequence, the power spectrum of the seasonally adjusted series $(x_t - s_t) = 0.25B^{-1}(1+B)^2 X_t$ is zero at the seasonal frequency. In this sense, the seasonal adjustment implies an "over-filtering" of the series. Furthermore, the filters involve future observations of the time series that are not available for recent values of the series. In practice the problem with missing future values is resolved by applying different filters at the end of the time series. For the last observation a one-sided filter is applied. However, this implies that the resulting series is no longer (difference) stationary even if the original series is.

From (7) and (8) it is seen that both *estimated* components are obtained by applying a linear filter to the series X_t. That is, although the theoretical model postulates that the components are driven by orthogonal errors u_t and v_t, the estimated components can be expressed as functions of the innovations $a_{t+1}, a_t, a_{t-1}, \ldots$. Thus, notwithstanding the different point of departure, the resulting components have quite similar properties.

Another problem with the unobserved component approach is that an orthogonal decomposition does not exist for a range of models. This is explained most easily in a simple model with stochastic trend. Let $X_t = p_t^* + \varepsilon_t$, where p_t^* is defined as before and ε_t is white noise with $E(\varepsilon_t) = 0$ and $E(\varepsilon_t^2) = \sigma_\varepsilon^2$. Furthermore, ε_t is uncorrelated with $u_t = \Delta p_t^*$, where $\Delta = 1 - B$. The first order autocorrelation of the differenced series is

$$E(\Delta X_t \Delta X_{t-1}) = -\sigma_\varepsilon^2.$$

Hence, the decomposition is not applicable if ΔX_t is a MA(1) process with positive autocorrelation. More generally, the decomposition requires that the spectrum of X_t has a global minimum at frequency zero (e.g. Watson 1986). A similar result holds for models with a seasonal component. Thus, an automatic application of the unobserved component approach appears problematical. By contrast, the BN-decomposition always exists provided that the time series has an appropriate ARIMA representation.

The structural time series model adopted, e.g., in SEATS is more complex than the decomposition defined in (6). In general, the decomposition can be represented as

$$X_t = p_t^* + s_t^* + u_t$$

where

$$\begin{aligned} p_t^* &= \frac{\beta(B)}{1-B} b_t \\ s_t^* &= \frac{\gamma(B)}{1+B} c_t, \end{aligned}$$

b_t and c_t are mutually uncorrelated white noise and $\beta(B)$, $\gamma(B)$ are polynomials in B. Identification of the parameters is achieved by *maximizing* the variance of the irregular component u_t.[5] The resulting representation is known as canonical decomposition (Hillmer & Tiao, 1983).

An important difference of this decomposition to the BN-decomposition is that the (pseudo) spectra of p_t^* and s_t^* depend on the parameters in $\beta(B)$ and $\gamma(B)$, while the spectral shapes of the nonstationary components of the BN-decomposition are the same for all time series. Accordingly, the trend and seasonal component of the canonical decomposition may possess important short term fluctuations which are not present when using the BN-decomposition.

4 The BN-decomposition in the frequency domain

From (2) and (3) it can be seen that the BN-decomposition implies the application of a particular linear filter so that the properties of the decomposition can be conveniently analysed in the frequency domain. For the ease of exposition we first discuss a trend model without seasonal component as in the original work of Beveridge & Nelson (1981). The effects in a model with seasonal component are straightforward.

Assume that the time series is generated by the model

$$(1-B)X_t = a_t - \theta a_{t-1}.$$

[5] An intuitive explanation for maximizing the variance of the irregular component is that the canonical decomposition aims at minimizing the noise of the predictable components and, hence, maximizing the variance of the irregular component.

The BN-decomposition yields

$$\begin{aligned} X_t &= (1-\theta)(1-B)^{-1}a_t + \theta a_t \\ &\equiv p_t + u_t \ , \end{aligned}$$

where $p_t = (1-\theta)(1-B)^{-1}a_t$ is the trend component and $u_t = \theta a_t = \Psi(B)X_t$ is a stationary component. The gain of the filter $\Psi(B)$ is given by

$$|\Psi(\omega)|^2 = \theta^2 \frac{2-2\cos\omega}{1+\theta^2-2\theta\cos\omega}.$$

Fig. 1 presents the gain function for various values of θ. Interestingly, for values of θ close to one, the gain function approaches the properties of an appropriate high pass filter. Setting $\theta = 1-\delta$ the gain function can be reformulated as

$$|\Psi(\omega)|^2 = (1-\delta)\frac{2-2\cos\omega}{(2-2\cos\omega)+\delta^2-2\delta(1-\cos\omega)}.$$

For small values of δ we have approximately

$$|\Psi(\omega)|^2 \approx \begin{cases} 0 & \text{for } \omega = 0 \\ 1 & \text{for } \omega > 0 \ . \end{cases}$$

Similarly, it follows that for small values of δ, the phase shift is approximately zero for $\omega > 0$. For $\omega = 0$ the phase shift is irrelevant because this frequency is (nearly) removed by the filter.

Fig. 1 and Fig. 2 about here

These results suggest that a simple and effective high-pass filter can be constructed by choosing a small value for δ, say $\delta = 0.1$. If the model is estimated, we can expect desirable properties of the BN-decomposition whenever θ is close to one, that is, if the process is *near integrated* in the sense of Phillips (1987). In fact, many economic time series seem to have this property and, consequently, unit root tests notoriously have difficulties to decide whether the time series are difference stationary or trend stationary. Thus, we can hope that the BN-decomposition renders filters with reasonable properties in many cases.

These results can be generalized, e.g., to quarterly models of the form

$$(1-B^4)X_t = (1-\theta B^4)a_t \tag{9}$$

As shown in the Appendix, the respective BN-decomposition is given by

$$X_t = \frac{(1-\theta)}{4(1-B)}a_t + \frac{(1-\theta)}{4(1+B)}a_t + \frac{(1-\theta)}{2(1+B^2)}a_t + \frac{(1-\theta)}{2(1+B^2)}a_{t-1} + u_t$$

where $u_t = \theta a_t - 0.5(1-\theta)a_{t-1} + 0.5(1-\theta)a_{t-3} - \theta a_{t-4}$ and the implied filter for removing the seasonal component results as

$$\Psi(B) = \frac{1 - 0.75(1-\theta) + 0.25(1-\theta)(B + B^2 + B^3) - \theta B^4}{(1-\theta B^4)}. \qquad (10)$$

Fig. 2 presents the gain function at different values of θ. As for the simple trend model, the filter has desirable properties if θ is close to one.

5 An Application

To compare the properties of the unobserved components approach (as implemented in SEATS) with the BN-decomposition, the German unemployment series running from 1962(i) to 1988(iv) is considered. This series is also analysed in Breitung (1994) and is selected to illustrate the main features of different approaches. Needless to say, that by considering a single example it is not possible to draw ultimate conclusions with respect to the merits and deficiencies of both approaches.

Applying the software package TRAMO to the original time series[6] the following model is selected and estimated:

$$(1 - 0.523B)(1 - B)(1 - B^4)X_t = (1 - 0.385B^4)\hat{a}_t \qquad (11)$$

Without employing any sophisticated model selection routine as in TRAMO, Breitung (1994) fitted an AR(4) model to differenced series $(1-B)(1-B^4)X_t$. The components extracted by the software SEATS are depicted in Fig. 3a–3c. It is interesting to note that the "irregular component" seems to be different from white noise. In fact, the estimated spectrum of this component given in Fig. 4 suggests that the irregular component behave different from a white noise series. This problem was already noted by Stier (1996, p. 319). Furthermore, it turns out that the irregular component is "over-filtered" in the sense that the power spectrum is zero at the seasonal frequencies.

Fig. 3 and Fig. 4 about here

Let $\psi(B) = (1 - 0.385B^4)(1 - 0.523B)^{-1}$ denote the MA polynomial of the differenced process. In the Appendix it is shown that the BN-decomposition

[6] The series is not transformed to logarithms or adjusted for outliers to facilitate the comparison with the BN-decomposition.

results as

$$
\begin{aligned}
X_t &= \phi_1(1-B)^{-2}a_t + \phi_2(1-B)^{-1}a_t + \phi_3(1+B)^{-1}a_t \\
&\quad + \phi_4(1+B^2)^{-1}a_t + \phi_5(1+B^2)^{-1}a_{t-1} + \psi^*(B)a_t
\end{aligned}
$$

where

$$
\begin{aligned}
\phi_1 &= \psi(1)/4 \\
\phi_2 &= -[\psi'(1) - 6\psi(1)/4]/4 \\
\phi_3 &= \psi(-1)/8 \\
\phi_4 &= \psi_R(i) - \psi_C(i) \\
\phi_5 &= \psi_R(i) + \psi_C(i) \ ,
\end{aligned}
$$

$i = \sqrt{-1}$, $\psi'(1)$ is the derivative of $\psi(\omega)$ evaluated at $\omega = 1$, $\psi_R(i)$ and $\psi_C(i)$ are the real and the imaginary part of $\psi(i)$, respectively.

Fig. 3d–3f presents the time path of the components resulting from the two-step estimation procedure of the BN-decomposition proposed in Breitung (1994). The trend component is quite similar to the respective trend component of the SEATS program. The seasonal component from the BN-decomposition reveals a more stable pattern than the seasonal component of the SEATS program.

The irregular component from the SEATS program and the transitory (or stationary) component of the BN-decomposition look quite different. However, this does not come as a surprise, since the irregular component should behave as white noise, while the transitory component from the BN-decomposition is a cyclical component that catchs up the spectral mass between zero and the seasonal frequencies.

The BN-decomposition provides a natural decomposition into trend (long-term) unemployment, seasonal unemployment, and cyclical (short-term) unemployment. It turns out that the recession of 1967 led to a sharp increase of cyclical unemployment, while the recession following the oil shock of 1973 yields a permanent increase in unemployment. The third recession (1982–1984) causes a sharp raise in cyclical *and* long-term unemployment.

Fig. 5 and Fig. 6 about here

Fig. 5 and Fig. 6 depicts the gain functions of the filters applied to remove the seasonal component. The general shape of the gain functions from the SEATS program (Fig. 5) and the BN decomposition (Fig. 6) look similar, however, the filter of the BN-decomposition emphasizes frequencies between $\pi/2$ and π. Consequently, the high frequencies of the seasonal component are dampened when using the filter of the BN-decomposition and a more stable seasonal component results.

6 Concluding Remarks

In this note, the unobserved component approach as implemented, e.g., in SEATS is compared to the decomposition resulting from a BN type of approach. It is argued that the unobservable component approach is conceptually more appealing but has a number of (theoretical) shortcomings which are not present when using the BN-decomposition. Specifically, (i) the orthogonal decomposition may not exist, (ii) it does not render a stationary series due to application of different filters at the end of the series, (iii) the seasonal components may suffer from substantial short run dynamics leading to a more variable seasonal pattern, (iv) the irregular component does not possess a flat spectrum, in general, and (v) the actual filter implies a kind of "over-filtering" in the sense that it introduces unit roots to the MA representation of the (differenced) series.

This does not mean, however, that the BN approach is generally preferable. In fact, it seems that the drawbacks of the orthogonal decomposition pointed out in this note are not very important *in practice.* For example, although there may not exist an orthogonal decomposition for the time series process, there may be a orthogonal decomposition that approximate closely the dynamic properties of the time series. Therefore, in practice it is wise to apply an approach with an appealing conceptual foundation and ignore potential problems with marginal practical relevance. In this sense, it is fine to have a well developed seasonal adjustment package like TRAMO–SEATS that can be used for routine application in statistical practice. Whether the BN technique turns out to be a promising alternative remains to be seen.

Appendix: The derivation of the BN-decompositions

Assume that X_t be generated by the process

$$(1 - B^4)X_t = \varphi(B)a_t \ ,$$

where $\psi(B)$ is a lag polynomial with all roots outside the unit circle of the complex plane. For $\psi(B) = (1 - \theta B^4)$ we have $\psi(1) = \psi(-1) = (1-\theta)/4$ and $\psi(i) = \psi(-i) = (1-\theta)/2$ so that from the results in Breitung (1994, p. 383) it follows

$$\begin{aligned}
(1 - B^4)X_t &= \frac{1}{4}(1-\theta)(1 + B + B^2 + B^3)a_t + \frac{1}{4}(1-\theta)(1 - B + B^2 - B^3)a_t \\
&\quad + \frac{1}{2}(1-\theta)(1 - B^2)a_t + \frac{1}{2}(1-\theta)(B - B^3)a_t + u_t \\
&= (1 - \theta B^4)a_t \ .
\end{aligned}$$

The process for u_t can be found from solving the equation for u_t.

Next, let X_t be generated by the process

$$(1 - B)(1 - B^4)X_t = \psi(B)a_t \ ,$$

where $\psi(B)$ is a lag polynomial with all roots outside the unit circle of the complex plane. The BN-decomposition is found from the expansion

$$\begin{aligned}
\psi(\omega) &= \phi_1(1 + \omega + \omega^2 + \omega^3) + \phi_2(1 - \omega^4) + \phi_3(1-\omega)^2(1+\omega^2) \\
&\quad + \phi_4(1-\omega)^2(1-\omega) + \phi_5(1-\omega)^2(\omega - \omega^2) + (1-\omega)(1-\omega^4)\psi^*(\omega).
\end{aligned}$$

where $\psi^*(\omega)$ is a polynomial with all roots outside the unit circle. Inserting $\omega = 1$ gives $\phi_1 = \psi(1)/4$.

The derivative evaluated at $\omega = 1$ gives

$$\psi'(1) = \left.\frac{\partial \psi(\omega)}{\partial \omega}\right|_{\omega=1} = 6\phi_1 - 4\phi_2$$

and, thus, $\phi_2 = -[\psi'(1) - 6\psi(1)/4]/4$. Inserting -1 gives $\phi_3 = \psi(-1)/8$.

Evaluating the expansion at $\omega = i$ and $\omega = -i$ gives

$$\begin{aligned}
\psi(i) &= 2\phi_4(1 - i) + 2\phi_5(1 + i) \\
\psi(-i) &= 2\phi_4(1 + i) + 2\phi_5(1 - i)
\end{aligned}$$

From this set of equations we get

$$\begin{aligned}
\phi_4 &= \psi_R(i) - \psi_C(i) \\
\phi_5 &= \psi_R(i) + \psi_C(i)
\end{aligned}$$

In our example we obtain

$$\psi(i) = \frac{1 - 0.385}{1 - 0.523i} = \frac{(1 - 0.385)(1 - 0.523i)}{1 - 0.523^2}$$

and, therefore,

$$\begin{aligned} \psi_R(i) &= \frac{(1 - 0.385)}{1 - 0.523^2} \\ \psi_C(i) &= 0.523\psi_R(i). \end{aligned}$$

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